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METHOD OF QUASI-GREEN'S FUNCTIONS FOR A NONSTATIONARY NONLINEAR PROBLEM OF THERMAL RADIATION

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We derive a system of two nonlinear integral equations for the determination of a temperature field and the intensity of the incident radiation. The kernels of these equations are expressed in terms of a quasi-Green's function.

One of the methods for increasing the accuracy of thermal calculations consists in converting a boundary value problem of heat conduction to an equivalent integral equation [1]. Various methods can be used for this purpose (see, for example, [2, 4]). In what follows, this conversion is effected with the aid of the method of quasi-Green's functions [5]. The main advantages of this method are: the explicit form of the kernels of the integrand expressions; the incorporation of information relating to the geometry of the domain of integration directly into the kernels using the apparatus of the theory of R-functions [6]. With an appropriate choice of structure for the normalized equation of the domain of integration [6], we obtain Fredholm integral equations of the second kind.

We consider a nonlinear initial-boundary problem for a heat radiating body in which the thermophysical characteristics and heat sources are temperature-independent and in which heat exchange with an external medium is present on a convex surface S (see [7]):

$$\operatorname{div}(\lambda \operatorname{grad} u) - c\rho u_t = -W, \quad P \in D, \; t > 0, \tag{1}$$

$$u(P, 0) = \psi(P), \quad P \in D, \tag{2}$$

$$\lambda \frac{\partial u}{\partial n} + \alpha u = \varphi(P, t, u), \quad P \in S, \; t > 0. \tag{3}$$

Here $\lambda = \Phi(\varphi, t)$ is the thermal conductivity coefficient; c is the specific heat coefficient; ρ is the density of the medium; W is the volumetric heat source or heat sink density,

$$\varphi(P, t, u) = -\varphi_0(P, t) + \varphi_1(P, t, u),$$

where $\varphi_0(P, t) = q_{source}(P, t) + \alpha u_m(P, t) + \varepsilon\sigma u_m^4(P, t)$ is the total heat flow supplied to S; $\varphi_1(P, t, u) = \varepsilon\sigma u^4$ is the flow radiated in accordance with the Stefan–Boltzmann law. Here u_m, in turn, is the temperature of the external medium; σ is the Stefan–Boltzmann constant; $\varepsilon = \varepsilon(u)$ is the degree of blackness of surface S.

If surface S contains a concave portion S_1 or if there is an exchange of radiative flows with other surfaces, then in the boundary conditions (3) an additional term $\varepsilon \times E$ appears in the function $\varphi_1(P, t, u)$ which accounts for radiation of heat on the concave surface S_1, and we then use the integral equations of radiant heat exchange

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$$E(P,t) = E_{source}(P,t) + \int K(P,Q)\{rE(Q,t) + \varepsilon F[u(Q,t)]\}\, dS_Q,\ P \in S_1. \tag{4}$$

Here E(P, t) is the integral hemispherical intensity of the incident radiation; E_{source}(P, t) is the external heat source intensity; r = 1-ε is the coefficient of reflection; K(P, Q) is a continuous, positive-definite, symmetric kernel:

$$K(P,Q) = \frac{\cos(P-Q,\ n_P)\cos(Q-P,\ n_Q)}{\pi(P-Q)^2},$$

P − Q is the vector joining points P and Q; n_P and n_Q are exterior normals to S_1 at points P and Q. Using Green's second formula for the operator Lu = div(λgradu)−cρu_t [7], we have

$$\int_0^{\tau+0}\int_D [vLu - uMv]\, dDdt = c\rho \int_D uv\Big|_0^{\tau+0} dD + \int_0^{\tau+0}\oint_S \left[v\left(\lambda\frac{\partial u}{\partial n} + \alpha u\right) - u\left(\lambda\frac{\partial v}{\partial n} + \alpha v\right)\right] dSdt, \tag{5}$$

where Mv = div(λ grad v) + cρv_t.

Substituting for v in Eq. (5) the fundamental solution of the thermal conductivity equation, namely,

$$v = r^*(P,Q,t-\tau) = \frac{1}{(2\sqrt{\pi/c\rho(t-\tau)})^3}\exp\left[-\frac{c\rho r_{PQ}^2}{4(t-\tau)}\right],$$

and making use of the following properties of the function r*(P, Q, t − τ) and the Dirac delta function,

$$Mr^* = -\delta(P-Q)\delta(t-\tau),\ P,\ Q \in D,\ t,\ \tau > 0, \tag{6}$$

$$\int_0^t\int_D f(Q,\tau)\delta(P-Q)\delta(t-\tau)\, dDd\tau = f(P,t), \tag{7}$$

we obtain

$$u(P,\tau) = -\int_0^{\tau+0}\int_D Lur^* dtdD - c\rho\int_D ur^*\Big|_0^{\tau+0} dD + \int_0^{\tau+0}\oint_S \left[r^*\left(\lambda\frac{\partial u}{\partial n} + \alpha u\right) - u\left(\lambda\frac{\partial r^*}{\partial n} + \alpha r^*\right)\right] dSdt. \tag{8}$$

Adding Eqs. (8) and (5), we have

$$u(P,\tau) = -\int_0^{\tau+0}\int_D [Lu(r^*-v) + uMv]\, dDdt +$$
$$+ \int_0^{\tau+0}\oint_S \left[\left(\lambda\frac{\partial u}{\partial n} + \alpha u\right)(r^*-v) - u\left(\lambda\frac{\partial(r^*-v)}{\partial n} + \alpha(r^*-v)\right)\right] dSdt - c\rho\int_D u(r^*-v)\Big|_0^{\tau+0} dD. \tag{9}$$

We construct the function v(P, Q) in the following form [5]:

$$v = v(P,Q,t-\tau) =$$
$$= \bar\omega(P,Q)\left\{\frac{\lambda(-c\rho)}{2(t-\tau)}\sum_{i=1}^{3}\left[(x_i-\xi_i)\frac{\partial\bar\omega}{\partial x_i} + (\xi_i - x_i)\frac{\partial\bar\omega}{\partial\xi_i}\right] + \alpha\right\}\frac{1}{(2\sqrt{\pi/c\rho(t-\tau)})^3}\exp\left(\frac{-c\rho[r^2 + 4\omega(x)\omega(\xi)]}{4(t-\tau)}\right), \tag{10}$$

where $r = \sqrt{\sum_{i=1}^{3}(x_i-\xi_i)^2}$; $P = P(x_1, x_2, x_3)$; $Q = Q(\xi_1, \xi_2, \xi_3)$; $\bar\omega(P,Q) = \omega(P)\Lambda^*\omega(Q)$; Λ^* is a symbol of R-conjunction, for example, $\Lambda^* = \Lambda_0$[6]; ω(x) is the normalized equation of the boundary of the domain of integration [5].

It is readily seen that with this choice for the function v, the corresponding function

$$G(P,\ Q,\ t-\tau)=r^{*}(P,\ Q,\ t-\tau)-v(P,\ Q,\ t-\tau) \tag{11}$$

satisfies the boundary condition

$$\lambda\frac{\partial G}{\partial n}+\alpha G=0. \tag{12}$$

Taking relations (1), (2), (3), and (12) into account, we see that Eq. (9) assumes the form

$$u(P,\ \tau)=U(P,\ \tau)-\int_{0}^{\tau+0}\int_{D}u(Q)\,Mv\,dD\,dt-c\rho\int_{D}u(Q,\ \tau)\,G(P,\ Q,\ 0)\,dD, \tag{13}$$

where

$$U(P,\ \tau)=\int_{0}^{\tau+0}\int_{D}WG(P,\ Q,\ t-\tau)\,dD\,dt+\int_{0}^{\tau+0}\oint_{S}G(P,\ Q,\ t-\tau)\ \varphi(P,\ t,\ u)\,dS\,dt+c\rho\int_{D}\psi(Q)\,G(P,\ Q,\ -\tau)\,dD.$$

Thus we have obtained a solving system of integral equations (13), also subject to the requirement (4), for the determination of the temperature field u(P, t).

The function $\omega(x)$ is constructed in a form which guarantees continuity of the kernel Mv of integral equation (13). Integral equations analogous to Eq. (13) can also be constructed for other boundary conditions.

NOTATION

u(P, t), temperature field of thermally radiating body; D, a finite region of three-dimensional space with convex boundary S; t, time; n, inner normal to boundary S; $r=\sqrt{\sum_{i=1}^{3}(x_i-\xi_i)^2}$, length of vector joining points $P(x_1,\ x_2,\ x_3)$ and $Q(\xi_1,\ \xi_2,\ \xi_3)$; S_1, concave portion of surface S.

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